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A LINEAR PROGRAMMING MODEL
FOR
THE ANALYSIS OF TRAFFIC IN A HIGHWAY NETWORK

A THESIS

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FOR

THE ANALYSIS OF TRAFFIC IN A HIGHWAY NETWORK

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CHAPTER I

INTRODUCTION

The purpose of this study was to develop the criteria necessary for the analysis of a congested urban highway network. As a result, a linear programming model is proposed that will distribute origin and destination volumes throughout a network in a theoretically optimum manner. The model simulates the optimum distribution of traffic in a highway network by minimizing the total travel time accumulated by peak hour drivers. Constraints limit the volume assigned to any given highway section to the capacity of the section and travel time is made a function of volume.

As a given limited-access facility becomes congested when demand exceeds capacity, special attention will be given to the effect of on-ramps, the only means of supplying traffic to the facility. The model should demonstrate under what conditions restrictive control of on-ramp volumes should be exercised during peak hours.

Intent of law.--Included in the Georgia Acts of 1955 is an act bringing the Georgia limited-access provisions into conformity with the standards of the interstate system. The "Declaration of Public Necessity" which was a foreword to this act states that because so many routes were becoming congested and the free movement of traffic impeded, the proposed system would be "designed and constructed to high standards and

for accommodating great volumes of through traffic in order to provide, maintain and protect these roads so as to preserve their traffic capacity."(1)*

Continuing congestion.--Certain portions of the limited access highways in the state are currently carrying such heavy volumes that the free movement of traffic is once again impeded and congestion is reaching the point where the traffic capacity of the routes is being diminished. Figure 1, derived from data gathered in a study of the North Freeway in Atlanta being conducted by the Georgia Tech Engineering Experiment Station, illustrates the speed-volume relationships for five minute intervals during a typical afternoon peak period. The shaded band illustrates the area of efficient movement as defined in this thesis.

Equilibrium and marginal costs.--The peak hour distribution of trips in an urban area is the result of the individual determination of best route by each motorist. In the resulting trip pattern no route will become extremely congested while an alternate route remains free flowing because drivers will adjust their paths, when possible, in order to avoid extreme congestion. In effect, a sort of equilibrium condition is reached. Any increase or decrease in the capacity of a given point in a network will result in a shift in path by the affected drivers, in order to gain time or avoid congestion until, once again, equilibrium is attained.

The question now arises as to whether this equilibrium condition arrived at by the value judgements of each driver is an optimum condition when all drivers are considered.

*Numbers in parenthesis refer to references listed under Literature Cited.

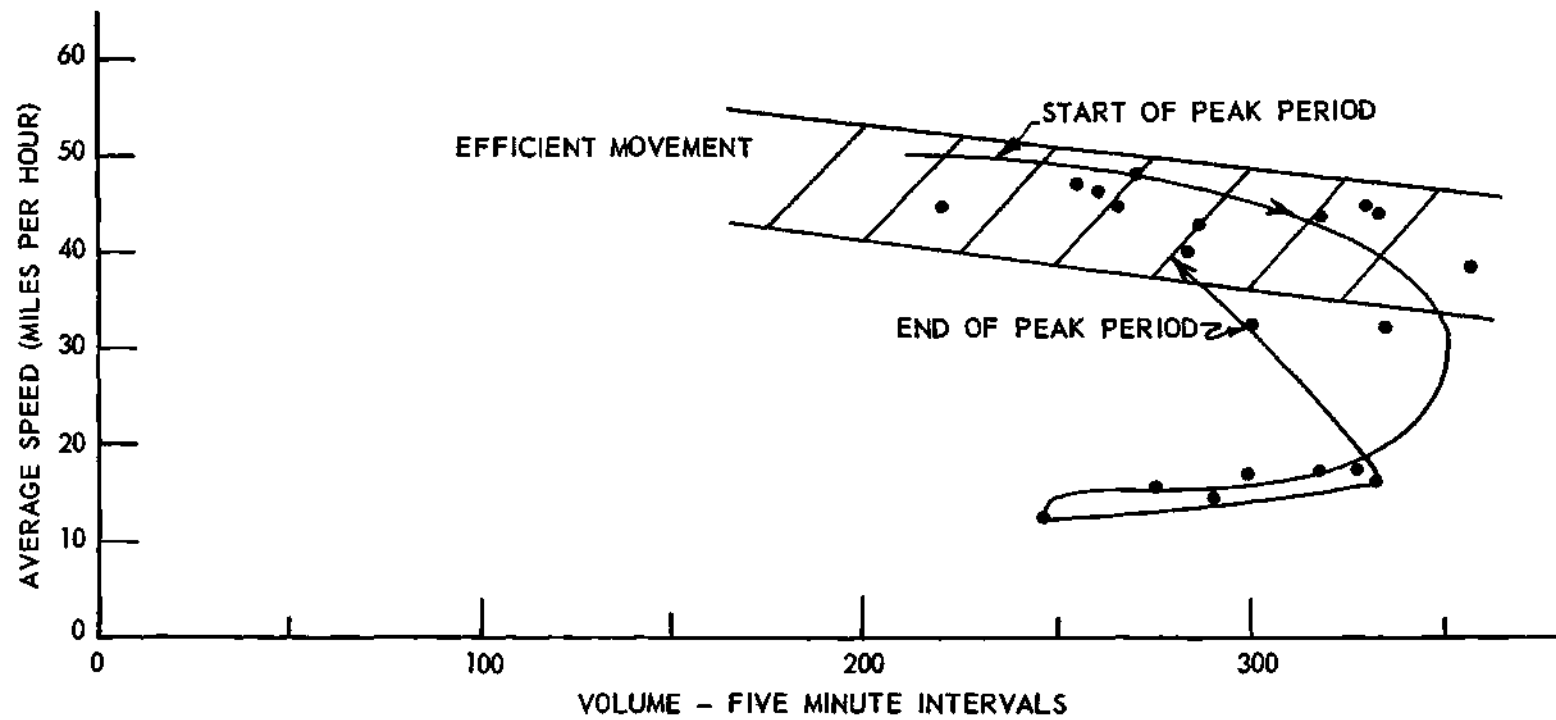


Figure 2 (derived from a figure in Freeway Operations, published by the Institute of Traffic Engineers (2)) illustrates the operation of a typical expressway. Using the upper limits of the curve illustrating the average maximum speed-volume relationships, it is seen that when an expressway is operating at 1400 vehicles per hour and maximum average speed attainable at this volume (40 miles per hour - 1.5 minutes per mile), the vehicles will accumulate 2100 vehicle-minutes of travel time in one mile. Now, if it is assumed that the volume increases to 1600 vehicles per hour, the speed will drop to 37 miles per hour and the time accumulated per mile is now 2600 vehicle-minutes. The addition of 200 vehicles per hour to the traffic stream has resulted in an increase of 500 vehicle-minutes of travel time for all vehicles, or if charged entirely to the added vehicles, 2.5 minutes per vehicle. Although the additional vehicles share of the total travel time is 2.5 minutes per vehicle, they can travel a one-mile section in 1.62 minutes. For these marginal vehicles the benefits (reduced travel time) are greater than the costs (share of the travel time).

This illustration is for a freeway flowing under ideal conditions. For a freeway that is experiencing severe congestion with a drop in speed to the point of capacity reduction, the marginal costs are very high.

To the off-peak user of the highway the problem is not serious. The situation does, however, become critical when marginal drivers accept breakdown of the facility, here defined as "stop and go operation." What is needed is an artificial constraint based on marginal costs that will prevent breakdown by limiting the volume of traffic permitted on a given facility.

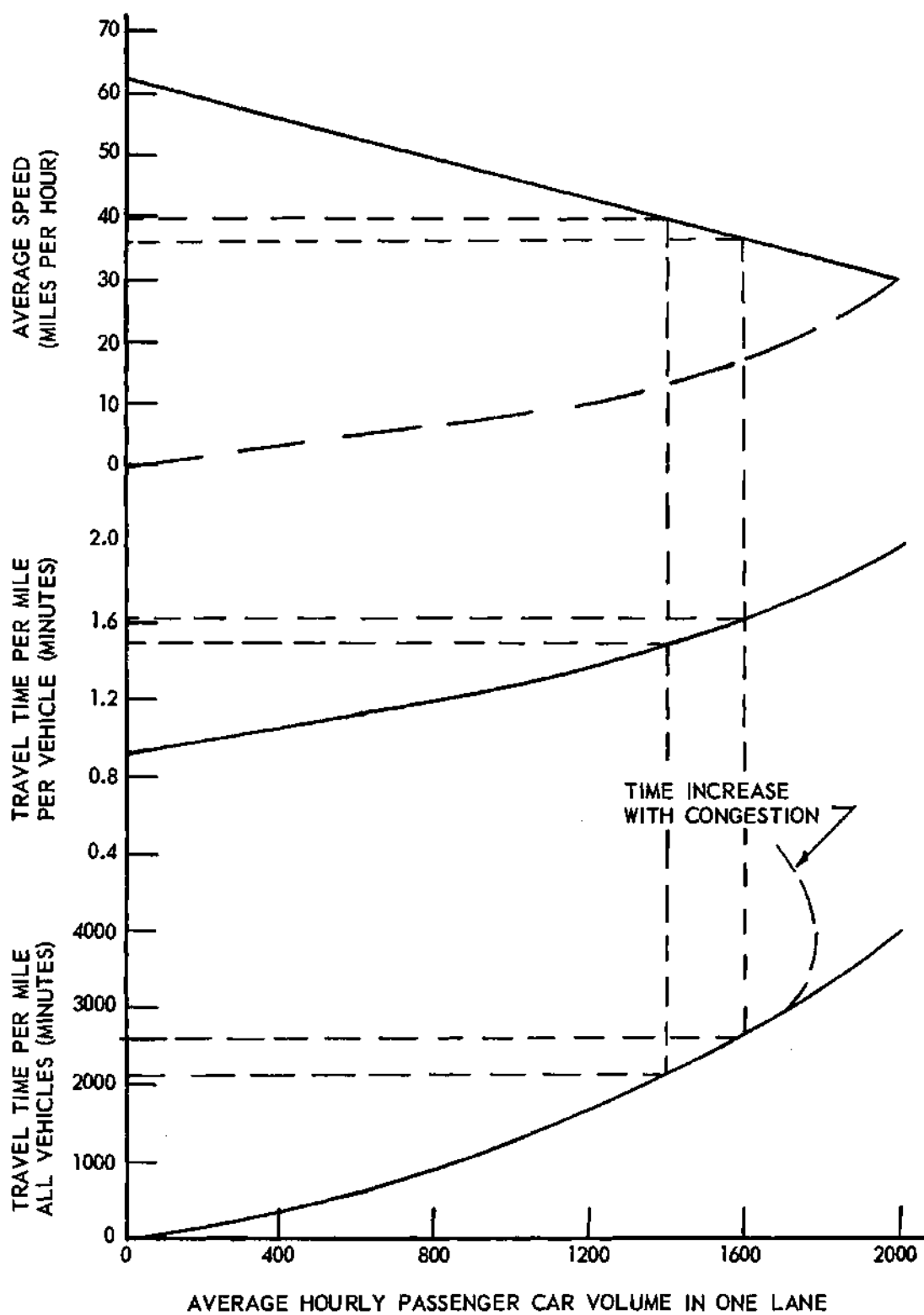


Figure 2. Travel Time on Four Lane Freeways.

CHAPTER II

EXAMINING THE SYSTEM

Constraints or Restrictive Controls

The history of traffic engineering contains many examples of the development of restrictive controls. The regulation and elimination of parking, prohibition of left turns, off-center lane movements, one-way streets, stop signs, and traffic signals are all examples of measures taken in order to gain maximum efficiency and capacity of streets and highways.

Need for regulation.--Individual drivers expect and respect efficient regulation. They know, for example, that when a traffic signal fails mechanically, congestion soon develops due to the lack of control. Yet, what is happening on many freeways? Closely spaced on-ramps, whose combined capacity vastly exceeds the capacity of the freeway itself, allow the volume of traffic on the facility to increase to the point of extreme congestion. Ramps that were designed to provide day-long convenient access to the freeway fulfill their design function for twenty-two hours, but in the remaining two hours they work as a detriment to the entire facility. Restrictive controls must be developed that will restore smooth flow.

Examples.--Some early limited-access facilities had on-ramps that were two lanes wide. As traffic volumes increased, it was necessary to stripe the ramps so as to allow only one lane useage. The changes were

necessitated by the combination of excessive friction caused by two lanes of vehicles attempting to merge simultaneously with the through traffic, and high accident experience due to poor sight distance. The result of the change was that while some traffic backed up on the ramps, the through vehicles were able to move more smoothly.

The most sophisticated example of restrictive traffic control is practiced by the Port of New York Authority in the operation of the Holland Tunnel. By limiting the volume of traffic entering the tunnel to a fixed number per minute, the Authority was able to eliminate the wave action which had resulted in stop and go operation. This action resulted in an increase in the total hourly flow in the tunnel. To quote the report, "it is believed that the Port Authority Project provides the first extensive controlled experimental verification of the concept that flow can be improved by being constricted."(3)

Measure of Effectiveness

Need for criteria.--In any highway network analysis certain parameters must be studied in order to evaluate the effectiveness of the system. Individual motorists are influenced in their choice of a route by travel time, travel distance, tolls, ability to keep moving, safety, convenience, habit, and possibly other factors.

If this were a study to determine whether to construct a given highway section, savings in travel cost vs. construction costs would have to be evaluated. The benefit-cost analysis has been developed to solve this problem.

Construction costs are not of concern in the current problem, but a form of benefit-cost analysis is necessary. Some drivers are going to

be penalized time and/or distance in the application of any restrictive control, while others are going to have better travel conditions. The measure of effectiveness will have to evaluate these changes in quantitative terms.

Selecting the correct criteria.--It has been stated that the trip distribution resulting from the decisions of the individual motorists is not optimum, but what constitutes an optimum distribution? The minimum time distribution, minimum distance, and a combination of these two factors utilizing a conversion to dollar values are all possibilities. It will be necessary to limit the discussion to these three factors as they are quantitative, which lends them to analysis, and more readily justified than such qualitative factors as convenience and habit.

Comparable Studies.--Studies to date in this area have approached the problem by trying to duplicate the existing travel pattern by isolating the correct criteria and then utilizing the information to assign traffic to a proposed highway network. Mainly, the studies have attempted to predict the traffic diverted from an existing arterial network to a new expressway.

While the studies vary in approach and results, their concensus is that time is the factor most values by motorists, with distance being rated somewhat lower in importance. Trueblood's study to determine why motorists were using the Shirley Highway, a limited-access facility, rather than the adjacent arterial highways showed that 81 per cent of the motorists saved time using the facility while only 38 per cent saved distance.(4)

Distance.--Distance alone falls short of being an ideal criterion, as it in no way recognizes the effect of congestion, which will be present to varying degrees in all urban highway networks.

Travel costs.--It would be possible to use a combination of time and distance as the measure of effectiveness by converting them to the same scale. Placing a monetary value on each is convenient, and this method is being used in most benefit-cost studies. For example, the California Highway Department uses the values shown in Table I for these studies.(5)

Table 1. Motor Vehicle Operating Costs (5)

	<u>Per Mile</u>	<u>Per Minute</u>
Passenger Cars, Freeways	4.5¢	2.6¢
Passenger Cars, City Streets	4.75¢	2.6¢
Trucks	14.0¢	5.0¢

Objectives.--Before selecting the criteria for the measure of effectiveness it is well to review the objectives of the study. They are:

1. To improve the quality of flow on a freeway by using restrictive control of ramp volumes in order to prevent congestion and thereby maintain high speed operation of the upper limits of the facility's capacity.

2. To ascertain that the net effect of the restrictive controls is positive by insuring that the time savings to expressway users will more than offset any time loss due to congestion occurring on arterial streets because of the ramp changes.

Time as the measure of effectiveness.--Since the key to the study is the elimination of congestion, it is felt travel time is the best indicator

of over-all effectiveness of an urban highway network. The addition of a distance factor, while possible, is not considered to be desirable as it would lessen the sensitivity of the model to congestion.

One should also remember that the aim of the model is not to duplicate an existing travel pattern, but is rather to arrive at an optimum feasible pattern. This points up the fact that the distribution determined by the model will be somewhat "artificial" and will require restrictive controls to accomplish.

Land Access

Right or privilege.--An Interstate route falls into the classification of a freeway, which is defined as a "divided arterial highway for through traffic with full control of access." (6) In making any decision to close a given ramp the question of the right of the people in the area served by the ramp to its continued use must be considered. First, it should be realized that the property owners have a "right" of access to abutting streets unless specifically denied by code or outright purchase of the rights. The use of any given street for passage of through vehicles is, however, a "privilege," subject to regulation by the authorities having jurisdiction for the purpose of regulating traffic.

Standards.--The position of the Bureau of Public Roads with regards to ramp spacings is best stated in the instructions for completing the estimate of cost for the Interstate System in accordance with Section 104(b)5, Title 23, U. S. Code, Highways. (7) The instructions state in part:

It is important that interchanges be located so as to properly discharge and receive traffic from other Interstate and Federal-aid system routes, or major arterial highways or streets. It is equally important that they not be spaced so closely as either to unnecessarily increase the cost of the system or interfere with the freeflow and safety of traffic on the Interstate System.

Interchanges within urban areas should not be spaced closer than an average of two miles, in the suburban sections of urban areas not closer than four miles, and in rural sections average not closer than eight miles.

Obviously, however, in consideration of the varying nature of the highway street or road systems with which the Interstate System must connect the spacings between individual adjacent interchanges must vary considerably. In urban areas the minimum distance between interchanges should not be less than one mile and in rural areas not less than three miles.

While exceptions have been made to the above-mentioned minimum spacings, every case must be individually justified on an economic basis.

Function of an on-ramp.--An on-ramp acts as a connector between the various classes of arterial streets which furnish access to abutting land and a freeway which has full control of access. In supplying traffic an on-ramp, in many instances, creates turbulence in orderly traffic flow. Drivers on a freeway will adjust their speed in order to retain the spacing that is being changed by the incoming vehicles. Some vehicles will move to the next inner lane in order to maintain their desired freedom of movement, and this in turn creates a disturbance in that lane. If another on-ramp is introduced at a point where the turbulence from the last ramp has not yet cleared, the problem will be magnified. As weaving or other conflicting movements take place in the traffic stream, speed drops, as drivers will reduce speed in order to satisfy their individual opinions of what constitutes safe driving conditions.

This situation illustrates the fact that there is a conflict between access and operation. Without good operation the value of access diminishes and it should not be too difficult to accept the fact that operation should receive the higher priority.

Legal provisions.--A review of the Georgia laws establishing limited-access highways clearly points out that the legislature intended that operation receive the priority over access. "Limited access highways may be designed as to regulate, restrict, or prohibit access thereto so as to best serve the traffic for which the facility is intended."(8) In addition, the authority of the highway department to make changes is stated. Highway authorities are "authorized and empowered to plan, designate, establish, regulate, abandon, alter, improve, and maintain limited access highways."(9) In discussing access points the law states that access will be provided only at the "designated points to which access may be permitted and under such arrangements and conditions as may be specified from time to time."(10)

The priority of through traffic is stated in the laws, along with the authority to make changes in access conditions. With travel time as the selected criterion, a rational analysis procedure for evaluating restrictive control of on-ramps must now be developed.

CHAPTER III

DEFINING THE PROBLEM

Nature of Congestion

Congestion defined.--Congestion is a relative phenomenon that may involve only a few vehicles or many. The time loss of the resultant delay varies greatly depending on the circumstances. Wingo (11) has defined congestion as "any situation between two or more vehicles such that one or more experience a time loss due to the behavior of the others." While this definition may be correct, it is difficult to apply, as it would label the most efficient operation of expressways (from a traffic engineering standpoint) as congested. For purposes of this presentation congestion will be defined as those traffic conditions under which the operating speed currently being attained is substantially below the maximum speed attainable corresponding to the given volume. For example, Figure 3 illustrates that it is possible for a two-lane portion of a divided highway to carry 3200 vehicles per hour at a speed of 37 miles per hour or, with increasing traffic density and restriction on movement, the section may carry the same volume at 18 miles per hour. Any movement below approximately 30 miles per hour would be termed congested.

The distinction between the definitions is that the first terms all conditions that involve restriction on movement (such as occur at maximum capacity) congestion, while the second requires that the speed of the restricted movement be less than optimum.

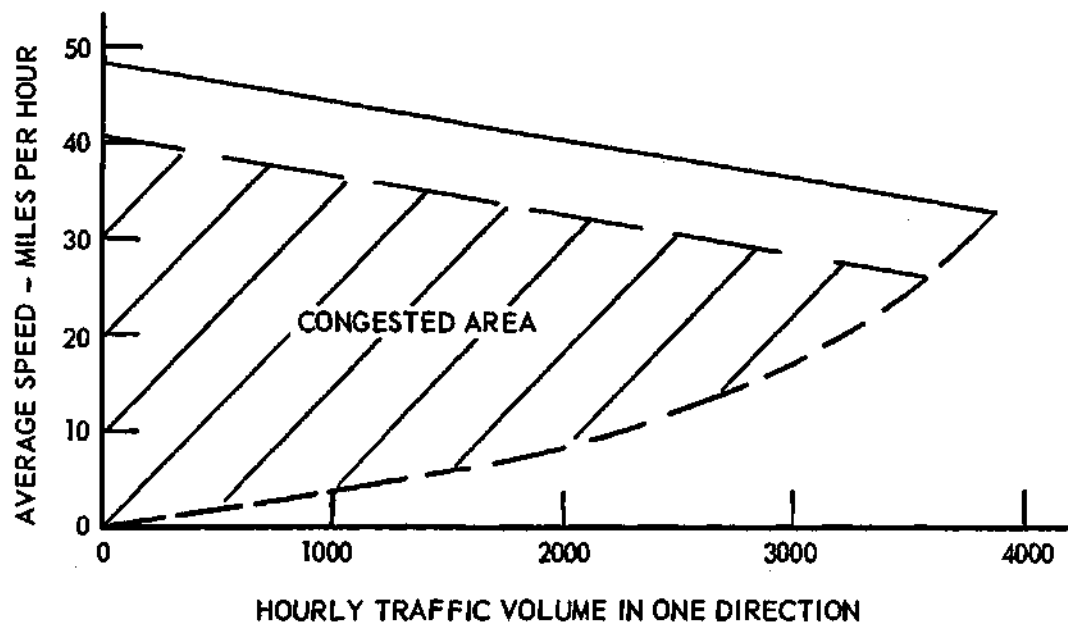


Figure 3. Congestion as a Function of Speed and Volume*.

*Adapted from Highway Capacity Manual, Fig. 7, p. 33.

Recently in the Journal of the Highway Division of the American Society of Civil Engineers there was a discussion on the merits of restrictive control of ramps.(12) The discussion centered mainly on the nature of congestion. Moskowitz referred to the results of an earlier study: "the volume during the hours when stop and go operation prevailed was at least as great as the volume ever obtained at the same location regardless of speed," to demonstrate that congestion does not mean breakdown of the facility.(13) The author of this thesis takes exception to this reasoning and defines breakdown as stop and go operation.

To fully understand the problem it is necessary to examine certain aspects of congestion. First, what causes congestion? The operating characteristics of drivers when they meet with given geometric and traffic conditions determine the capacity of a facility. When demand for a particular section of roadway exceeds its capacity, congestion will result.

Capacity defined.--Because capacity is also relative, certain definitions have been standardized in order to improve highway evaluation procedures.(14)

Possible capacity - the maximum number of vehicles that can pass a given point on a lane or roadway during one hour under the prevailing roadway and traffic conditions.

Practical capacity - the maximum number of vehicles that can pass a given point on a roadway or in a designated lane during one hour without the traffic density being so great as to cause unreasonable delay, hazard, or restriction to the drivers' freedom to maneuver under the prevailing roadway and traffic conditions.

Basic capacity - the maximum number of passenger cars that can pass a given point on a lane or roadway during one hour under the most nearly ideal roadway and traffic conditions which can possibly be obtained.

Relation of capacity to congestion.--Congestion occurs somewhere in the region between practical and possible capacity, depending on varying factors of the given day such as weather and percentage of trucks. The problem of congestion resulting from accidents or vehicle breakdowns will not be considered, as it involves point congestion while this study is concerned primarily with congestion as it affects an entire network.

The capacity of a highway facility is not everywhere the same. Conditions such as an adverse grade which will slow trucks, a short radius curve, or a heavy weaving movement will all tend to reduce the capacity of a given point on the facility. If the volume of traffic approaching the point of restricted capacity is greater than the capacity of the restricted point, congestion will result.

The volume of traffic leaving the restricted portion may be the same as the volume it could adequately carry without congestion, but because of the congestion every driver will have to undergo a period of stop and go operation. The result is that although the same maximum number of vehicles may be progressing through the point, every driver is experiencing some delay and the facility is, in effect, serving as a storage area supplying vehicles at a given rate (one every two seconds per lane) to the restricted portion of the facility.

Congestion Illustrated

Point congestion.--The following illustration (Figure 4) is a convenient method of demonstrating the type of congestion resulting from demand in excess of capacity. With vehicles clearing the restricted point plotted as the ordinate and time as the abscissa, a cumulative plot describes the

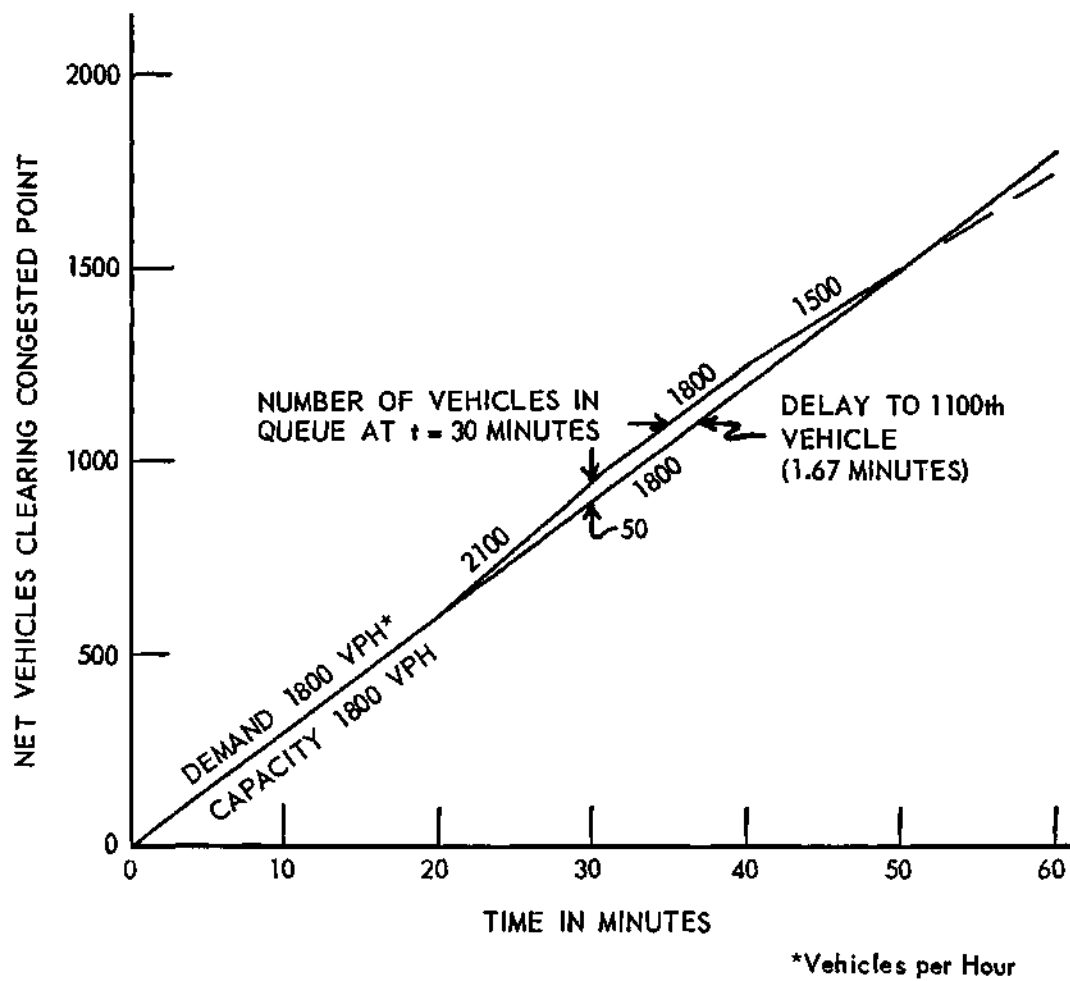


Figure 4. Congestion Illustrated.

condition of the system throughout the time when congestion is occurring. The area enclosed by the demand and the capacity curves equals the total delay to all vehicles. In this illustration a hypothetical one-lane highway has a capacity of 1800 vehicles per hour. For the first twenty minutes, demand just equals capacity and no congestion results. For the next ten minutes the demand exceeds the capacity by 300 vehicles per hour and congestion develops. At $t = 30$ minutes there are 50 vehicles in the queue behind the restriction and the delay to each vehicle is 1.67 minutes. For the next ten minutes the volume again equals the demand but the magnitude of the congestion remains the same. At $t = 40$ minutes the demand falls below the capacity and the congestion begins to clear.

Congestion with ramp involved.---The same approach will now be utilized to demonstrate the phenomenon of congestion caused by an on-ramp on a freeway already flowing at capacity. (Figure 5) Assume that at $t = 0$, the two-lane portion of a four-lane divided freeway is flowing at a rate of 3800 vehicles per hour, its possible capacity. For forty minutes when the freeway is flowing at capacity the on-ramp has a demand of 4000 vehicles per hour. The freeway demand then equals the capacity for the next twenty minutes before dropping to 3000 vehicles per hour at $t = 80$ minutes.

The upper portion of the figure demonstrates what occurs if the ramp traffic is permitted on the freeway. During the period of peak congestion there are 266 vehicles involved with each vehicle experiencing four minutes of delay. Note that both through vehicles and ramp vehicles are involved.

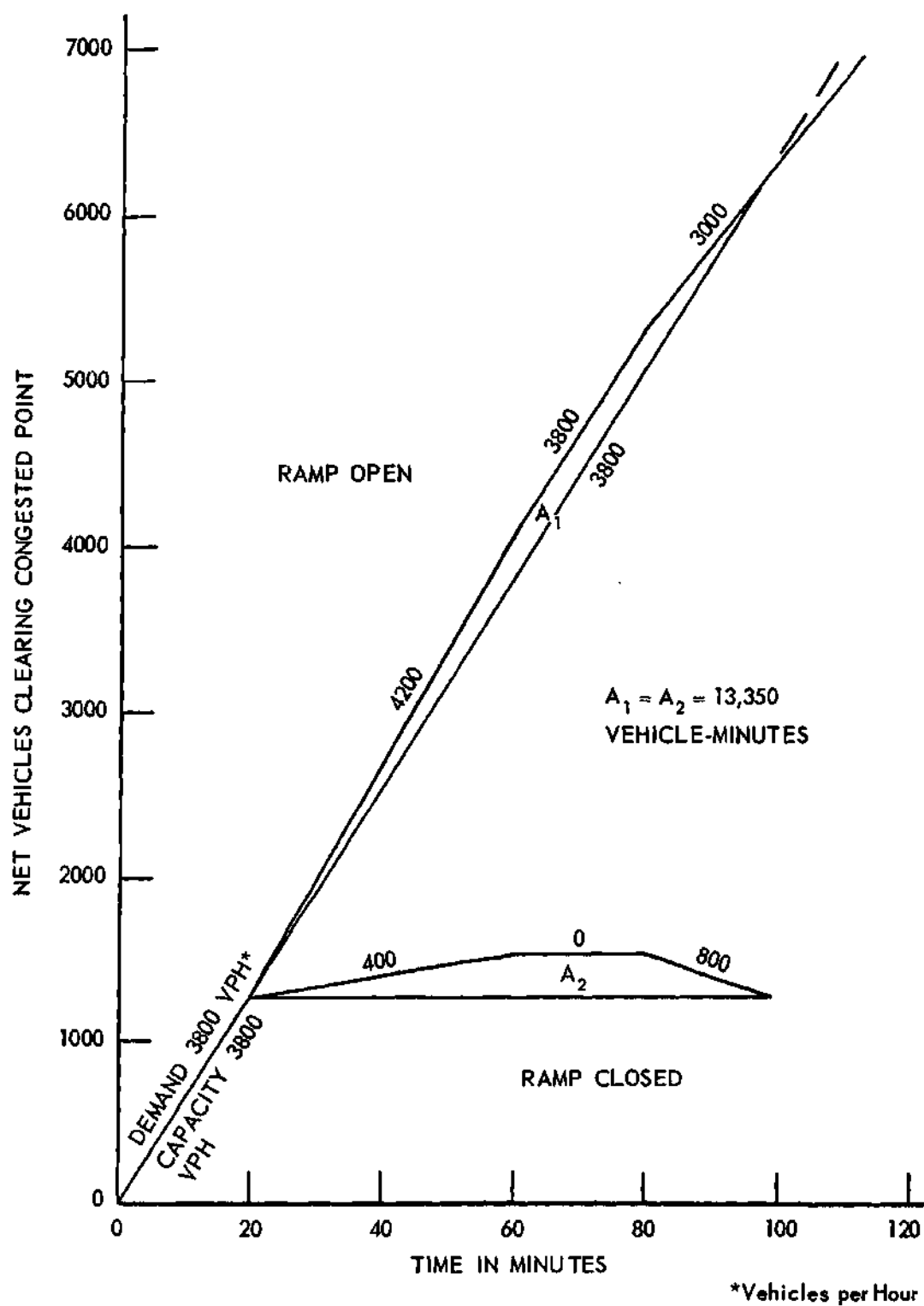


Figure 5. Congestion at an On-Ramp.

The lower portion of the figure demonstrates what occurs if the ramp is closed to traffic while the freeway is flowing at capacity. At the peak period of the congestion 266 ramp vehicles will be stopped and the average time of involvement will be forty-eight minutes. During this time the expressway traffic will be flowing unimpeded.

In the first instance the delay is distributed among all the vehicles, both on the freeway and those desiring entrance from the ramp. In the second instance the delay is experienced only by the ramp vehicles. Note that the area enclosed by the demand and capacity curves, representing total vehicle-minutes of delay, is the same in both instances.

The point that this figure is designed to demonstrate is that while drivers will probably accept the four minutes of delay in the first instance, the ramp drivers in the second instance will not wait forty-eight minutes to enter the freeway, but rather will take another path that does not involve such a high fixed delay. The net result is that the total travel time accumulated by all vehicles may be substantially reduced.

This illustration vastly simplifies the true situation exhibited at any given location, but it does demonstrate a valid relationship that is not readily apparent in a field study.

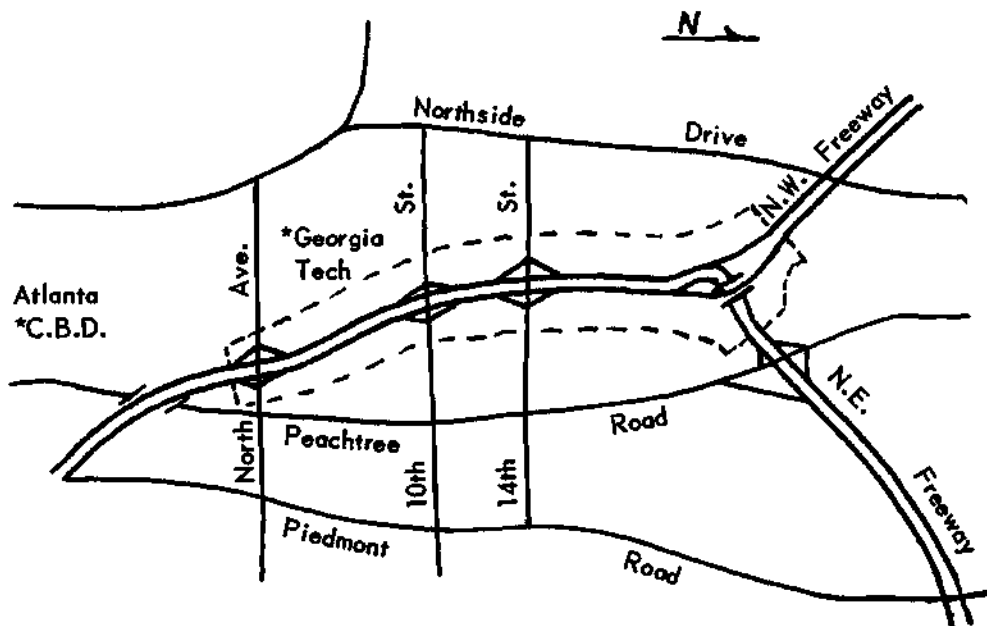
Capacity Problems

Control points.--What is capacity? In the last section the accepted definitions of capacity were given. These definitions were all stated in terms of the capacity of a point on a lane or highway. However, on an urban freeway the capacity of a given point is only significant if

the point is a control point. By control point is meant a point that forms the lower capacity limit of a freeway section. In an urban highway network there are many control points and in making an assignment it is difficult, but necessary, to consider all of them.

Excessive number of ramps.--An additional aspect of capacity is shown in Figure 6. Examine the diagram of the North Freeway in Atlanta, along with its volume-capacity curves. The volumes represent a typical weekday evening peak hour operation. Note that the volume builds up until it just equals the capacity at Fourteenth Street. However, given a fairly stable demand distribution with near constant ramp volumes, there is good reason to state that the freeway is operating at capacity throughout its entire length. Although there are gaps for accepting more volume at both North Avenue and Tenth Street, any increase in volume at these points will result in congestion at Fourteenth Street, because the demand will have exceeded the capacity. Any congestion at Fourteenth Street will soon back up to the other interchanges and the entire section will operate under stop and go conditions until the demand begins to decrease. This situation illustrates that urban network capacities must be examined in terms of geometrics, ramp spacings, and flow patterns.

Ramps at congested locations.--A second condition under which ramp volumes are critical is the situation where an on-ramp is attempting to supply vehicles to a freeway that is already flowing at capacity. To a certain extent any freeway that is flowing at capacity will exert its own restrictive control on the on-ramp volumes, as there will be few gaps in the oncoming stream for vehicles to enter. However, a field examination of freeway flow under these conditions will show that within a short



LOCATION SKETCH OF NORTH FREEWAY, ATLANTA

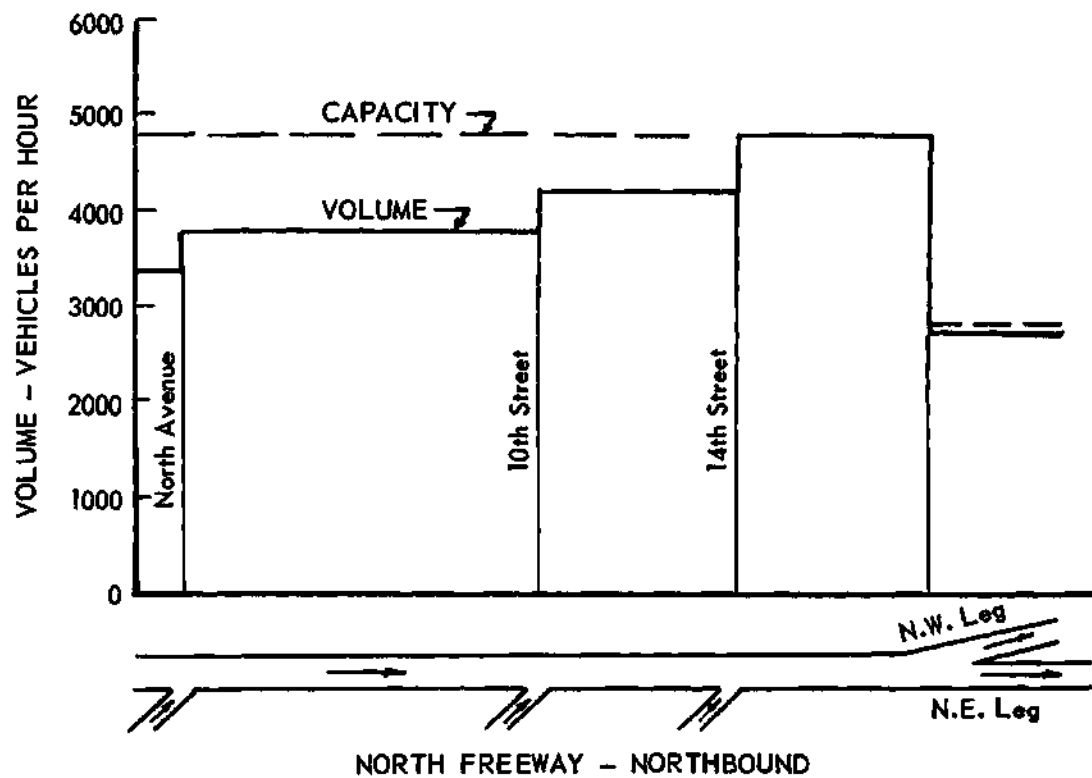


Figure 6. Volume and Capacity Along the North Freeway During a Typical P.M. Peak Hour.

period of time a ramp driver will attempt to enter an insufficient gap, necessitating a drastic reduction in speed on the part of the driver at the rear of the gap and in turn the vehicles behind him. There is also, on occasion, a freeway driver who will stop to let ramp vehicles enter. In the resultant slowdown it is likely that capacity will also decrease. At certain times it has been necessary to resort to police control of ramp terminals on the North Freeway in order to prevent complete stoppage of traffic.

Figure 7, taken from the Highway Capacity Manual, illustrates the relationship between volume on an expressway and the possible capacity of an on-ramp.(15) In a later section it will be shown how this relationship of ramp capacity to expressway volume may be incorporated as a requirement of the linear programming model.

Special conditions.--When a ramp is located in relation to another interchange in such a manner that a heavy, restricted weaving movement is occurring because of the predominant travel pattern, special controls may be necessary. This condition will not normally occur when recommended ramp spacings have been followed.

Summary.--The three conditions under which a ramp may act as an impediment to the efficient operation of a freeway are:

1. The combined volume of adjacent ramps is so large, because of close spacing or high demand, that congestion is occurring at some point ahead of them.
2. Interference with freeway traffic is occurring at the ramp terminal because the freeway is already operating at capacity.

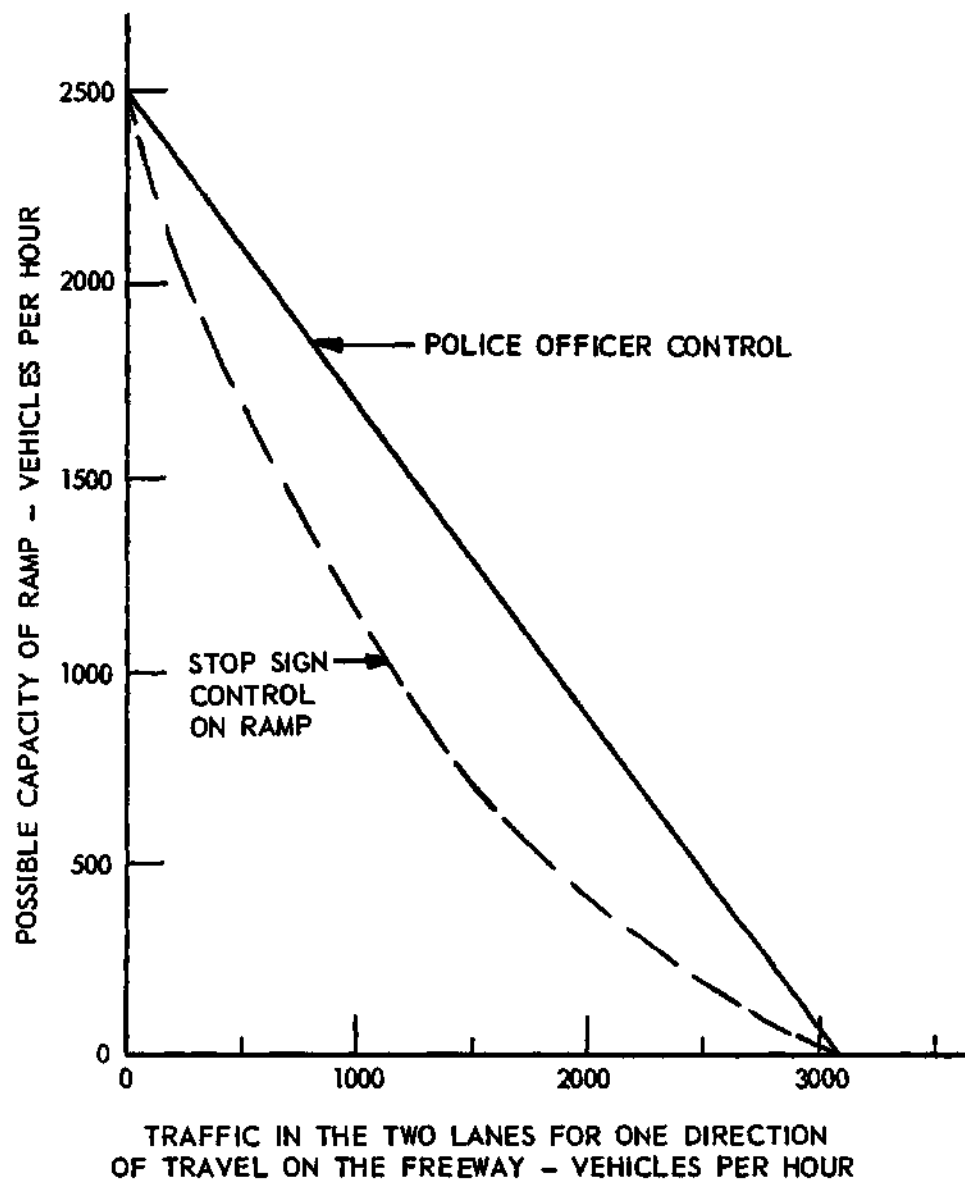


Figure 7. Variation in Ramp Capacity with Volume of Traffic on an Expressway⁽¹⁵⁾.

3. A ramp is located in such a manner in relation to an interchange ahead that a restricted weaving movement is decreasing the capacity of the freeway.

Equilibrium and Efficiency

Equilibrium defined.--In the introduction the statement was made that the distribution of traffic in any urban network is basically an equilibrium problem. In the classical economic sense a condition of equilibrium implies that, given two routes between two distant points, one shorter but of limited capacity and the other longer and of unlimited capacity, motorists would use the shorter route until the time that the congestion costs on the shorter route would make it more economical to use the longer route. At this point the average cost of transportation may be said to be the same for either route. If under these conditions a few vehicles could be induced to leave the congested route, it would be possible to greatly lessen the costs of driving this route while only slightly increasing the costs of driving the longer uncongested route. This condition arises because the marginal costs for the congested route are much higher than those of the uncongested route. For example, the addition of a few vehicles to a freeway operating at possible capacity may, because of congestion, cause a drop in capacity along with a drop in speed. If these same vehicles were diverted to a freeflowing route, they would have an insignificant effect on the other vehicles.

Efficiency must be guaranteed.--A problem arises in the analysis of urban routes in that the diversion of a small volume of freeway traffic may constitute a significant volume in terms of arterial street capacity.

There should be some guarantee that a given ramp restriction will not be merely chasing rainbows, that is, improving conditions at one point only to have serious congestion develop at another previously trouble free spot.

It is for this reason the highway system must be considered a network with many possible routings available to the motorist. This is analogous to the true situation, but the difficulty arises in the mathematical representation of this complex situation.

Are congestion costs marginal?--There may be some question as to the marginal costs of increasing congestion. As the Highway Capacity Manual points out, "The relative restriction and the average travel time will increase uniformly as the traffic volume on a highway is increased from low volumes to the maximum possible capacity of the highway." (16) A reexamination of Figure 2 will show that although the increase in travel time is uniform, it is not a straight line increase. Speed decreases linearly with increasing volume, but at the same time the increase in travel time is exponential. Figure 2 represents ideal conditions with no congestion. However, in the upper regions of a highway's capacity, any factor which will cause a slowdown in traffic will result in congestion and a sharp upswing in the travel time curve (dashed line bottom).

CHAPTER IV

SOLUTIONS TO THE PROBLEM

Types of Analysis

Atlanta study.--Currently underway on the North Freeway in Atlanta is a study being conducted by the Georgia Tech Engineering Experiment Station.* The purpose of the study is to develop criteria for ramp spacings. An experiment has been carefully set up whereby certain ramps are closed for two week periods. When a traffic pattern is established, measurements are made of speed, volumes, and travel times, throughout the network. Upon completion of the fieldwork the data will be analyzed in order to determine the relative merits of each situation and also to determine if there is a significant difference in the operation of the freeway under the different ramp conditions. In effect there will be six discrete cases each with its own travel pattern and aggregate travel time.

California study.--A field study was also conducted on a California freeway. In this study travel times and trip paths were obtained for many vehicles in the network. By analyzing the data it was possible to determine which vehicles were involved in congestion. The effect of closing a given ramp was then determined by deduction.(17)

*Project HPS-1(59), Georgia Tech Engineering Experiment Station, "The Influence of Ramp Spacing on Traffic Flow on the Atlanta Expressway."

Need for base data.--Field studies such as the two just mentioned will provide adequate solutions to the problems of malfunctioning ramps. They have, however, some drawbacks. They are very expensive and time consuming; and if ramps are closed experimentally, the travel patterns of several thousand motorists will be disrupted.

At the present time field studies must be made in order to develop base data for more advanced techniques. No synthetic study could be made without the data developed from extensive field studies. However, once an adequate amount of base data has been developed, simulation techniques may be used to advantage.

Simulation.--Another possible procedure is mathematical simulation of network conditions. Although much work has been done on micro-analysis of traffic conditions through simulation, the complications involved in the simulation of flow on a network basis are still overwhelming. There is no adequate theory of traffic flow and monte carlo techniques on a network basis are beyond the capacity of available computers. This is not to say that a solution will not soon be available using these techniques, as much work is being done in this field.

Much success has been obtained in the assignment of given volumes to a traffic network on the basis of minimum time or distance paths. Most of these methods do not take into consideration the capacity of the links involved, although there have been recent developments in this area.(18)

The Linear Programming Model

Linear Programming.--The model developed in this study makes use of linear programming, a mathematical optimizing technique developed in the

last decade. In linear programming, a linear objective function may be optimized subject to a series of linear constraints. The term "linear" simply means that if it takes one minute to go one mile it will take ten minutes to go ten miles. The restriction of linearity limits the application of the technique but a review of many problems shows that its field of application is very broad and that many complex problems may be restated or approximated in linear terms.

Mathematical background.---This model is designed to use the simplex technique, a computational procedure for solving the general linear programming problem. In the general linear programming problem a mathematical model or description of the problem is stated using linear or straight line relationships. Mathematically these statements are of the form:

$$a_1x_1 + a_2x_2 + \dots + a_jx_j + \dots + a_nx_n = a_0$$

where the a_j 's are the known coefficients (time) and the x_j 's are the unknown variables (volume). The complete mathematical statement of a linear programming problem involves a set of simultaneous linear equations which represent the conditions of the problem and a linear function which expresses the objective of the problem (minimize aggregate travel time).

Characteristics of the model.---The conditions or constraints of the model reflect the desirable operating characteristics of the network. Before presenting the model some of its features will be discussed.

Basically the model assigns trip desires to alternate paths in a manner that will minimize the total travel time accumulated by all vehicles. It has two primary constraints:

1. The total volume assigned to any link must be less than the capacity of the link.
2. The sum of the fractional parts of a given trip desire assigned to alternate paths must be equal to the total desire.

In addition, special controls are assigned to make the capacity of a ramp a function of the traffic already on the through facility. Special treatment is also applied to the constraints so that the travel time accumulated by the vehicles approximates the time distribution observed on operating facilities. That is, the higher the volume the lower the speed up to capacity. In the model the lower speed corresponds to a higher travel time.

Explanation of the Model

Definitions.--

1. Objective function - a linear combination of the variables to be optimized by the selected solution. In this model the total travel time accumulated by all vehicles is to be minimized.
2. Node - a point of trip origin, destination, or interchange.
3. Link - a connection between two nodes representative of a highway facilities.
4. Path - a series of connected links representative of a trip route.
5. Network - the combination of all links and nodes.

Notations.--

V_{12} - total desire between node 1 and node 2.

1-2 - link connecting node 1 to node 2.

V_{12A} - portion of V_{12} going via path A.

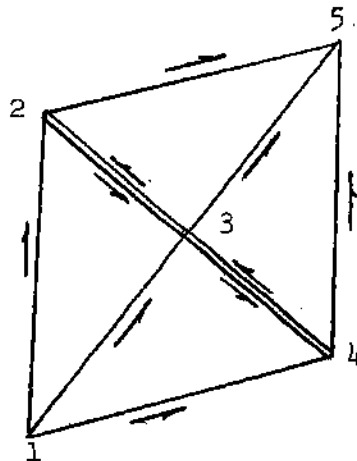
c_{12} - practical capacity of link 1-2.

K_{12} - practical-possible capacity increment of link 1-2.

t_{12A} - basic travel time per vehicle via path 12A.

t'_{12A} - added travel time per vehicle between practical and possible capacity via path 12A.

Example.--For means of illustration a simple network consisting of five nodes and ten directional links is presented.



Consider that the following are the significant desires during the peak hour: V_{12} , V_{14} , V_{15} , V_{24} , V_{42} . First examine V_{12} . This trip could be made by traveling either 1-2, or by going by way of 3, 1-3-2. There are many other possible paths but these are considered the only feasible paths. Similarly, the other desires have alternate travel paths.

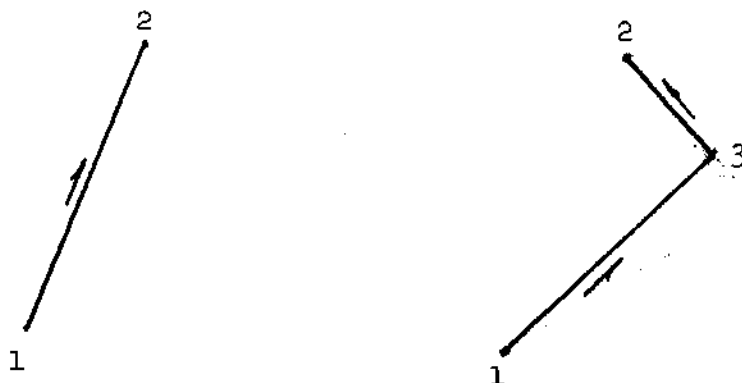


Table 2 is a convenient form of listing the variables prior to writing the simplex equations. Desires are listed vertically, and links horizontally. Note that for each desire the alternate feasible paths are listed. For example, V_{12} has two alternate paths V_{12A} and V_{12B} . V_{12A} uses only one link and therefore it is noted only under link 1-2. V_{12B} uses two links and it is noted under 1-3 and 3-2. Figure 8 summarizes all of the constraint equations.

Constraints.--The first constraint is that the sum of the fractional parts of a given trip desire assigned to alternate paths be equal to the total desire. Hence for V_{12} :

$$V_{12A} + V_{12B} = V_{12}$$

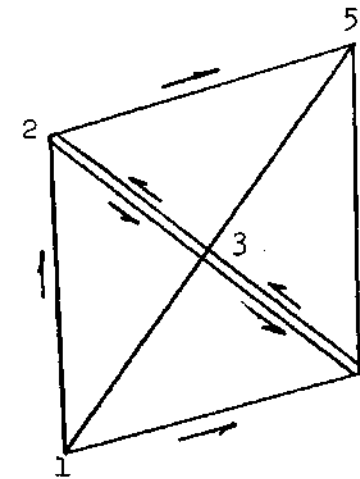
In each case any one of the fractional desires may equal zero, the total desire, or any value in between.

The second constraint is that the total volume assigned to any one link be less than or equal to the capacity of the link. Referring to the table, link 1-3 would have the following constraint equation:

$$V_{12B} + V_{14B} + V_{15A} \leq c_{13}$$

Table 2. Sample Problem Formulation

	Links									
	1-2	1-3	1-4	2-3	3-2	3-4	4-3	2-5	3-5	4-5
Desires	V_{12}	V_{12A}	V_{12B}		V_{12B}					
	V_{14}		V_{14A}			V_{14B}				
	V_{15}	V_{15A}						V_{15A}		
		V_{15B}	V_{15C}					V_{15B}		V_{15C}
	V_{24}			V_{24A}		V_{24A}				
	V_{42}				V_{42A}		V_{42A}			



Sketch of Network

Practical Capacity	c_{12}	c_{13}	c_{14}	c_{23}	c_{32}	c_{34}	c_{43}	c_{25}	c_{35}	c_{45}
Marginal Capacity	K_{12}	K_{13}	K_{14}	K_{23}	K_{32}	K_{34}	K_{43}	K_{25}	K_{35}	K_{45}

Fractional parts of desires sum to total desire.--

$$V_{12A} + V_{12B} = V_{12}$$

$$V_{14A} + V_{14B} = V_{14}$$

$$V_{15A} + V_{15B} + V_{15C} = V_{15}$$

$$V_{24A} = V_{24}$$

$$V_{42A} = V_{42}$$

Volume is less than capacity.--

$$V_{12A} + V_{15B} - K_{12} \leq c_{12}$$

$$V_{12B} + V_{14B} + V_{15A} - K_{13} \leq c_{13}$$

$$V_{14A} + V_{15C} - K_{14} \leq c_{14}$$

$$V_{24A} - K_{23} \leq c_{22}$$

$$V_{12B} + V_{42A} - K_{32} \leq c_{32}$$

$$V_{14B} + V_{24A} - K_{34} \leq c_{34}$$

$$V_{42A} - K_{43} \leq c_{43}$$

$$V_{15B} - K_{15} \leq c_{15}$$

$$V_{15A} - K_{35} \leq c_{35}$$

$$V_{15C} - K_{45} \leq c_{45}$$

In addition, every K (Marginal capacity) must be bounded.--

$$K_{12} \leq \text{increment}$$

$$K_{13} \leq \text{increment}$$

.....

$$K_{45} \leq \text{increment}$$

Figure 8. Restraint Equations for Sample Problem

However, at this point the constraint must be modified to take into consideration the marginal time increase as volume increases. The equation now is:

$$V_{12B} + V_{14B} + V_{15A} - K_{13} \leq c_{13}$$

In effect this constraint says that the total volume assigned to link 1-3 must be less than or equal to the possible capacity of the link, with a "time penalty" added for every vehicle over the practical capacity.

The following example illustrates the manner in which the constraint works. Consider a highway section with the following characteristics:

practical capacity = c_{12} = 1600 vehicles per hour

practical-possible increment = K_{12} = 400 vehicles per hour

∴ possible capacity = 2000 vehicles per hour

travel time at practical capacity = 4.0 minutes

travel time at possible capacity = 5.0 minutes

Using the following constraint as an example:

$$(\text{link 1-2}) \quad V_{12A} + V_{15B} - K_{12} \leq c_{12} \text{ or } 1600$$

Assume that the total capacity of the link is split evenly between the two desires. If the total demand is 1600 vehicles per hour, K_{12} will equal zero, and $V_{12A} = V_{15B} = 800$. The total time accumulated by the vehicles is $1600 \times 4 = 6400$ vehicle-minutes.

Now consider that the demand is high enough so that all the capacity is used.

$$V_{12A} + V_{15B} - 400 = 1600$$

and

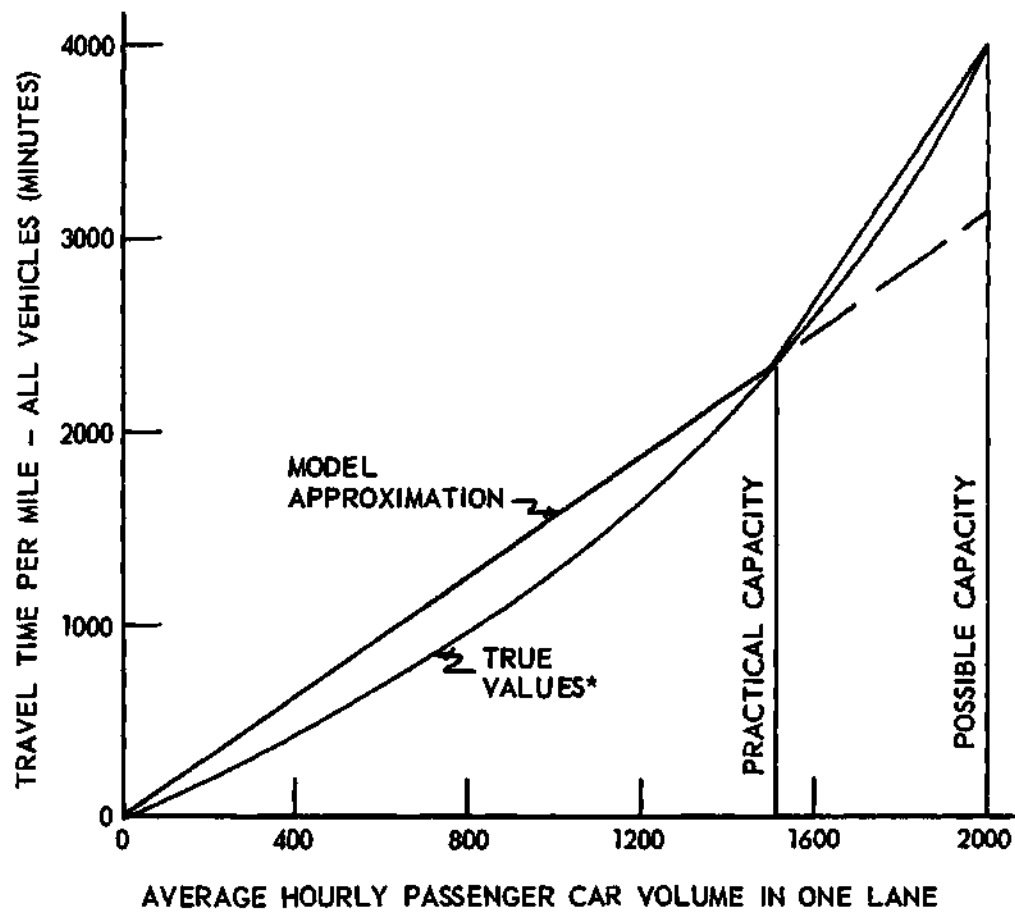
$$V_{12A} = V_{15B} = 1000$$

Assuming that the time it takes to travel the given section increases from four to five minutes per vehicle with the increase in volume, the total time accumulated is now $2000 \times 5 = 10,000$ vehicle-minutes.

In the model the mathematics are as follows. All vehicles are assessed the four minutes travel time per vehicle up to possible capacity, or $2000 \times 4 = 8000$ vehicle-minutes. This leaves a 2000 vehicle-minute penalty to be assessed to the last 400 vehicles or five minutes per vehicle. This value (5 minutes per vehicle) is therefore assessed to every unit of "K", the practical-possible capacity increment. In effect, the last 400 or marginal vehicles have been charged for the slowdown to the first 1600 vehicles. Their net travel time is 9 minutes per vehicle.

Figure 9 demonstrates graphically the assumptions just stated. The curve, adapted from Figure 2, illustrates the true increase of vehicle-minutes, while the straight lines demonstrate the model's approximation of the true conditions. The best fit is in the upper regions of capacity and one should realize that this is the area which will be utilized during the peak hours.

The selection of practical capacity as the point where the assessment of the added penalty is begun is somewhat arbitrary, but the volume above practical capacity does represent the trouble zone during peak



*Developed from Figure 2.

Figure 9. Travel Time Accumulation.

flow conditions. The Highway Capacity Manual describes the region between practical and possible capacity as "a reservoir which can absorb an overload with increasing inconvenience to drivers."(19)

The objective function.--For the sample problem the objective function is the summation of the volume of each fractional desire multiplied by its travel time, plus the volume of the practical-possible capacity increment used multiplied by its share of the travel time as determined in the simple calculation previously illustrated.

$$t'_{ab} = \frac{(V_1 \times T_1) - (V_2 \times T_2)}{V_1 - V_2}$$

where t'_{ab} equals the assigned travel time for K_{ab}

V_1 equals the possible capacity

T_1 equals the travel time at possible capacity

V_2 equals the practical capacity

T_2 equals the travel time at practical capacity

The equation for the objective function for the sample problem is as follows:

$$\begin{aligned} &V_{12A} \times t_{12A} + V_{12B} \times t_{12B} + \dots \\ &+ K_{12A} \times t'_{12A} + K_{12B} \times t'_{12B} + \dots = \text{Total travel time} \end{aligned}$$

The solution to the problem is reached when:

$$\sum (v_{ab} \times t_{ab} + K_{ab} \times t'_{ab}) \text{ is a minimum.}$$

At this point it is well to point out that linear programming is merely a very efficient method of arriving at the optimum solution. With the simplex method the first step involves finding a solution that satisfies all the constraints, even though it is not the optimum solution. Additional solutions are then determined in a computationally efficient manner. An examination of the summation of the objective function for each solution would show that each one has a value less than or equal to the summation of the previous solution. No solution is even attempted that does not represent an improvement on the previous solution.

A convenient rule of thumb is that it will take two iterations for every constraint in the original problem. This is a very small number when it is compared with the almost infinite number of feasible solutions in a large problem.

Data needed for practical applications.--The following is a summary of the minimum amount of data needed to apply the suggested procedure.

1. The network of arterial streets and expressways that are to form the basis for the model must be specified.
2. The peak hour demand as represented by origin and destination desire volumes must be known. The desires should be grouped so that only the significant volumes in the direction of peak hour flow are considered. Here, a balance should be obtained between the accuracy of the model and available storage in the computer.
3. The practical and possible capacities of each link should next be determined. On arterial streets the capacities should represent the intersectional capacity that governs the particular section.

4. The average speeds should next be determined, representative of both practical and possible capacity. The speeds should then be converted to travel times for the given section lengths.

5. The final step involves the selection of alternate feasible paths for each trip desire. The selection should begin with the minimum time path with alternate paths consisting of the routes a motorist might take if he were to become involved in congestion on the minimum time path. Most computer programs for solving the linear programming problem are arranged in such a manner that the number of alternate paths that may be specified is, for all practical purposes, unlimited.

A small pilot study to establish the speed-volume relationship for the various classes of streets and highways is all the field work that should be necessary to form a sufficient basis for the model.

Possible modifications.--Any situation involving the interaction of vehicles that may be expressed in linear terms may be included as a constraint in the model. For example, earlier it was mentioned that there is an interaction between ramp volumes and expressway volumes. While it has not been possible to demonstrate this interaction in linear terms with data available locally, Figure 9, taken from the Highway Capacity Manual, will be used as an illustration.(p. 28)

The equation of the boundary conditions may be stated as
 $0.83X + Y = 2500$, or in terms of a simplex constraint:

$$0.83(V_f) + V_r \leq 2500$$

where V_f is the volume on the freeway prior to the ramp

V_r is the volume entering from the ramp.

For this constraint it is not necessary to specify the ramp as a link as long as a time penalty is not added for the ramp itself. If a time penalty is desired, a "K" factor may be added to the equation.

The same procedure may be applied to weaving volumes. For example, a weaving section 1800 feet in length has a weaving capacity of 3200 vehicles per hour at an operating speed of 30 miles per hour.(20) For a given section the desires which must weave can be noted and the following constraint written:

$$W_1 + W_2 = 3200$$

$$\text{or } (V_{ab} + \dots) + (V_{cd} + \dots) \leq 3200$$

where W_1 equals the volume weaving to the right.

W_2 equals the volume weaving to the left.

In the same manner left turns or any other significant traffic feature may be constrained or penalized time.

A highway network may be clearly described in mathematical terms by this series of linear constraints. The problem now is to ascertain that the technique will determine a flow pattern that is not only "theoretically" optimum, but that is also capable of attainment through restrictive control of selected ramp volumes.

The optimum distribution.--Consider a highway network with uniform speed and unlimited capacity. If a linear programming model were applied to this network, every vehicle would be assigned to its minimum time path. This is the same distribution that the so called "all or nothing"

conventional assignment technique would determine. For many unknown reasons, usually lumped together under the heading of human behavior, drivers, as social beings, do not always choose the minimum time path. The "diversion curve" assignment technique, by assessing the relative merits of the best path using the freeway as well as the best path using the arterial streets, assigns a certain portion of the desire volume to each route. In this manner a closer approximation of the actual field conditions is attained.

Without an opportunity to apply the model, it appears that the actual results will lie somewhere in between the two methods. This statement is made because, although the model attempts to predict an optimum condition, at the same time it recognizes capacity constraints and to a degree, congestion. Some desires will be assigned all to one path while others will be split between many routes. As was stated earlier, the number of feasible trip distributions is very large. Because the model recognizes this factor and at the same time has realistic parameters, the optimum solution should provide a realistic approximation of efficient network conditions.

Additional Features

Arterial street capacity.--Up until now "K," the practical-possible capacity increment, has been assumed to be bounded. To allow travel time to determine the upper bound for "K" on the arterial streets is feasible and perhaps desirable. By using a very high time value for "K," congestion is effectively simulated. There is no problem in using this approach for arterial streets, as traffic signals operate at their

highest capacity under loaded cycles. This approach should not, however, be used for freeways as it is more than likely that with increasing congestion there will be a decrease in capacity.

Figure 10, which shows the delay at signalized intersections, illustrates the intended relationship.(21) The dashed lines indicate the linear approximation of the curve. A three-stage linear approximation could be used just as easily, but the accuracy of the model would probably not be significantly improved.

A steep slope must be used on the second stage of the time-volume curve when "K" is unbounded. This relationship is necessary, for the point at which high travel time would make it more feasible to use an alternate path should conform approximately to the true capacity. If this rule is not followed, the model will, in effect, stretch out the service time for traffic to more than an hour and the basis for the model, that is during the peak "hour," will have been destroyed.

Loading pattern.--By setting a definite limit on the capacity of the freeway and, in effect, duplicating the true increase of travel time on the arterial streets, a very desirable relationship is established.

Consider what would happen if the model were applied to successively increasing demands. With very low volume all traffic would be assigned to its minimum time path, with the expressways being favored because of their high running speed. As the traffic volume increases and the marginal travel time of vehicles above practical capacity takes effect, the arterial streets begin to be favored. As traffic increases still further to the point at which an analysis of this type becomes

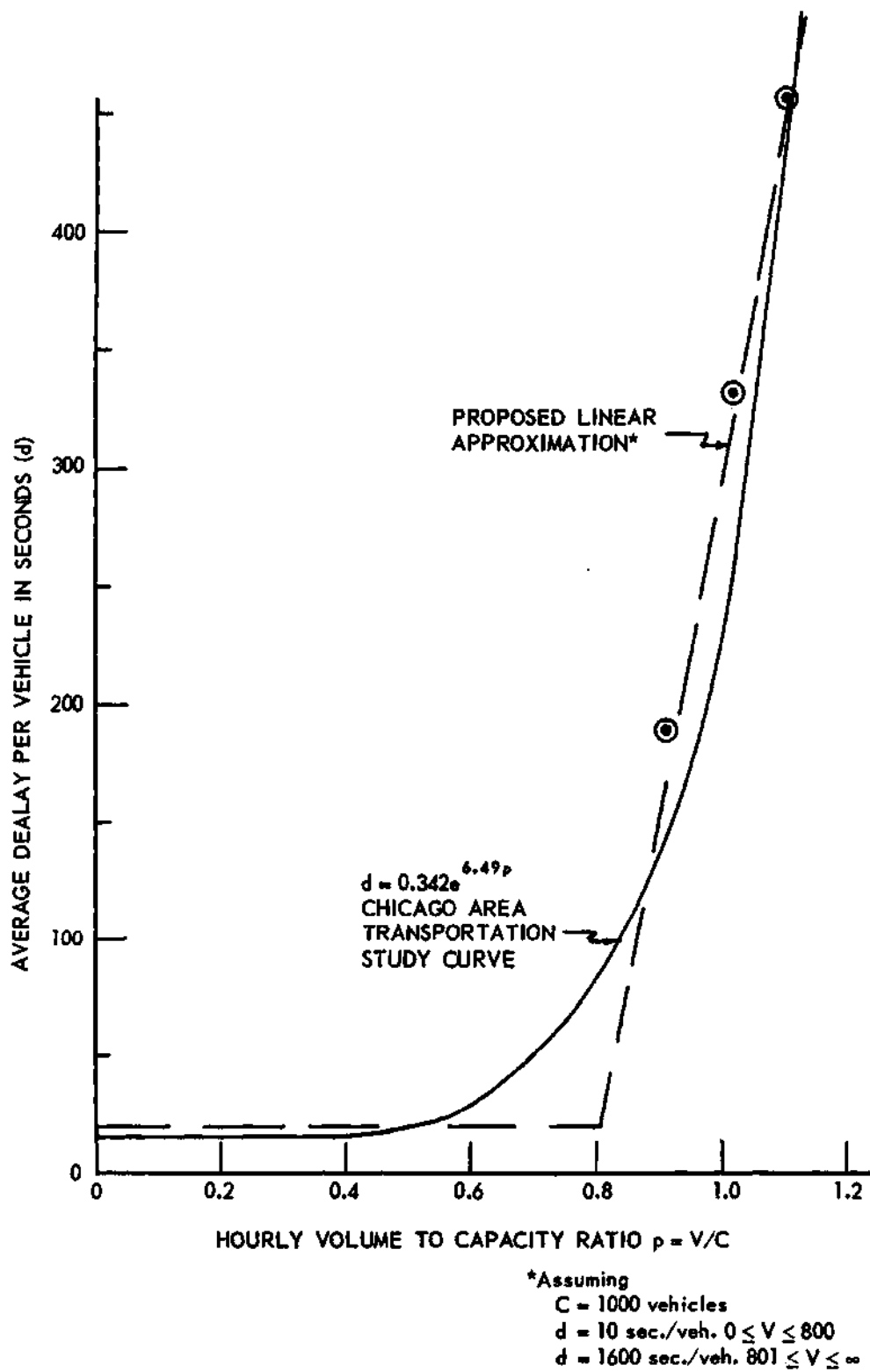


Figure 10. Delay at Signalized Intersections⁽²¹⁾.

advisable, a form of balancing begins. The marginal time penalties of the freeways are compared with the congestion costs of the arterial streets. Because no overloading is allowed at any point on the freeway system, its capacity will not begin to decrease due to congestion. The result is that ideal freeway conditions are compared with realistic arterial street conditions.

The optimum solution will therefore indicate those points where the freeway is currently being overloaded and at the same time reveal the locations where improvements to the arterial streets are warranted.

Computer programs.--Gass, in his text on linear programming, lists fourteen different programs that were available in 1958 for solving general linear programming problems.(22) Programs are listed for ten different computers and approximately half of them are large enough to handle a meaningful highway network application. The IBM 704 has the most versatile program listed, having a capacity of 255 constraints and unlimited unknowns. The method used is the revised simplex, utilizing the product form of the inverse and upper bound techniques. The basic computer needed for use with this program is the IBM 704 with 4 logical drums, 4 tapes, and 4096 words of magnetic core storage.(23)

Size of a highway network problem.--In general, the number of restraint equations required will be equal to the summation of: (1) the number of desires, (2) twice the number of freeway links, and (3) the number of arterial links.

A constraint equation is required for each desire in order to keep the trip input from varying. Two constraints are required for each link for which a practical and a possible capacity have been specified. One

equation will define the capacity-travel time relationship when an arterial link is left unbounded. When additional features such as a weaving or turning restriction are utilized, one equation will be required in each instance.

Figure 11 illustrates the network consisting of the North Freeway in Atlanta, Georgia, and the adjacent arterial streets for which the suggested analysis was designed. The network consists of 64 directional links with 16 points of origin and destination. The following breakdown of constraint equations applies to this network:

8 freeway links	16 constraints
56 directional arterial links	56 constraints
5 ramps	5 constraints
1 restricted weave	<u>1 constraint</u>
Total Network Constraints	78 constraints

A full origin and destination table for the 16 points where trip ends are recorded would consist of 240 directional desires, if the intrazonal trips, which are non-assignable, are neglected. More than one-third of the 240 desires are eliminated from consideration as they are not in the direction of peak hour movement. An examination of the desire table would show that there are 130 desires that are feasible in the peak hour direction. The desires eliminated are mainly to stations 1-5, the downtown area. These volumes are insignificant when compared with the outbound flow and no links are specified to consider them.

Allowing one equation for each of the 130 desires and 78 to define the network, the total number of constraint equations required to determine the minimum time distribution of trips for this network is 208. This figure is well within the capacity of 255 constraints of the IBM 704 program.

Transportation problem.--There may be some question as to why the "transportation method," a modification of the simplex technique, was not used. Even though no coefficients are used on the basic constraint variables, other limitations preclude its use. Unbounded links connecting each point of origin directly with each point of destination are normally required. Orden (24) has developed the transshipment method to allow shipment through intermediate points, but unbounded connecting links are still required.

The most significant drawback is the specialized input and output of a highway network. When dealing with commodities located at different markets, if the selling prices and shipment costs are known it is not a difficult problem to determine how much of the commodity should be purchased from each market in order to minimize the costs of fulfilling your requirements. With a highway network the trip ends are fixed in number and location and every trip must be made. The variable is the trip path. Because of the complexities of specifying this situation the more versatile simplex technique must be used.

Buses and restrictive ramp control.--Although not directly concerned with the model, one topic worthy of mention in connection with any anticipated ramp closing is the operation of commuter buses. Consider the case where because of congestion on the outskirts of the downtown area closing one of the downtown ramps is required to decrease the volume at the point of congestion. Under these conditions it would be very desirable to allow operation of commuter buses on the ramp. One of the reasons for the failure of bus operation on freeways to gain more popularity is the

problem of ramp congestion which contributes a disproportionate share of the bus trip time. By allowing only buses to operate on the ramp under these conditions the bus riders would have a much faster trip and automobile travelers would be unaffected.

CHAPTER V

RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

Results.--The following items form a statement of the results of the thesis.

1. Travel time was determined to be the best indicator of overall efficiency of an urban highway network.
2. Considering both legal and operational aspects of the problem, land access should be a secondary consideration when restrictive control of ramp volumes is anticipated.
3. The concept of equilibrium with the determination of the marginal time increment as volume increases provides an excellent procedure for evaluating the use of restrictive control of ramp volumes.
4. Because of current computer limitations the model applies to networks up to a size of approximately 100 links. As many as 155 desires may be assigned to a network of this size. For each desire an unlimited number of paths may be specified.
5. There is an appreciable amount of work involved in developing the model; however, once the model has been verified it could become a subroutine of a more extensive computer procedure.

Conclusions.--

1. Linear programming provides an effective method for determining the minimum time distribution of traffic in an urban highway network.

2. The restrictive control of ramp volumes may be a justifiable procedure that should be utilized to help prevent freeway congestion.

3. Freeway operation under stop and go conditions is, in most instances, uneconomical with respect to time, because the minimum aggregate travel time for peak hour traffic is not being obtained when a small number of vehicles are allowed to cause serious congestion.

4. The model presented is not intended for use on a metropolitan area basis because of current computer limitations, but rather it may prove to be a satisfactory alternative to a field study when the operating characteristics of a portion of an urban highway network are to be studied in the hopes of improving traffic flow.

5. Conventional assignment techniques do not provide a satisfactory approach to operational problems of an existing highway network unless they incorporate capacity restraint measures.

6. The linear programming model logically provides a satisfactory approximation of actual traffic conditions. A practical application is warranted in order to verify and improve the suggested procedures.

Recommendations.--

1. Additional field studies should be undertaken in order to develop the parameters necessary for the application of mathematical procedures to traffic analysis.

2. The linear programming model should be applied to an urban highway network that is experiencing severe congestion in order to verify and improve the suggested procedures.

LITERATURE CITED

1. Georgia Laws 1955, Act No. 333, Limited-Access Highways, p. 560.
2. Freeway Operations, Institute of Traffic Engineers, Washington, D.C., 1961, p. 11.
3. Edie, Leslie C., Foote, Robert S., "Tunnel Traffic Flow," Proceedings, Highway Research Board, Vol. 39, 1960, p. 502.
4. Trueblood, Darel L., "Effects of Travel Time and Distance on Freeway Usage," Highway Research Board Bulletin 61, 1952.
5. Schmit, Robert E., Campbell, M. Earl, Highway Traffic Estimation, The Eno Foundation for Highway Traffic Control, Saugatuck, Conn., 1956, p. 141.
6. Highway Capacity Manual, U. S. Department of Commerce, Bureau of Public Roads, 1959, U. S. Government Printing Office, p. 9.
7. Warren, W. L., "Interchanges," California Highways, Vol. 40, May-June 1961, p. 32.
8. Georgia Laws 1955, op. cit., p. 560.
9. Ibid., p. 561.
10. Ibid., p. 562.
11. Wingo, Lowdon, "Measurement of Congestion in Transportation Systems," Highway Research Board Bulletin 221, 1959, p. 2.
12. Barnett, Joseph, "Operations of Urban Expressways," Journal of the Highway Division, American Society of Civil Engineers, Paper 1374, Sept. 1957, discussions by Moskowitz, Karl, and DeLeuw, Charles E., May 1958, closure by Barnett, Oct. 1958.
13. Webb, G. M., Moskowitz, K., "California Freeway Capacity Study," 1956. Proceedings, Highway Research Board, 1957, p. 613.
14. Highway Capacity Manual, op. cit., pp. 6-7.
15. Ibid., p. 125.
16. Ibid., p. 39.

17. Brenner, Robert, Telford, Edward T., Frischer, Donald, "A Quantitative Evaluation of Traffic in a Complex Network," Paper presented at the Highway Research Board, Annual Meeting, Jan. 1960.
18. Mertz, William L., "Evaluation of Electronic Computer Traffic Assignment Programs," Unpublished report to Origin and Destination Survey Committee, Highway Research Board, Annual Meeting, Jan. 1961.
19. Highway Capacity Manual, op. cit., p. 8.
20. Ibid., p. 115.
21. Haikalis, George, Joseph, Hymen, "Economic Evaluation of Traffic Networks," Paper presented at the Highway Research Board, Annual Meeting, Jan. 1961, p. 26.
22. Gass, Saul I., Linear Programming, McGraw-Hill, New York, 1958, p. 132.
23. Orchard-Hays, William, Judd, Hal, Cutler, Leola, "Linear Programming," Nov. 1, 1956, SHARE Distribution #161, IBM Applied Science Special Problems Group.
24. Orden, Alex, "The Transshipment Problem," Management Science, Vol. 2, 1956, pp. 276-285.

OTHER REFERENCES

1. Beckmann, Martin, McGuire, C. B., Winsten, Christopher B., Studies in the Economics of Transportation, Yale University Press, New Haven, 1956.
2. Garrison, William L., and Marble, Duane F., "Analysis of Highway Networks: A Linear Programming Formulation," Proceedings, Highway Research Board, Vol. 37, 1958.
3. Irwin, N. A., Dodd, Norman, von Cube, H. G., "Capacity Restraint in Assignment Programs," Unpublished report to Origin and Destination Survey Committee, Highway Research Board, Annual Meeting, Jan. 1961.